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Pareto optimization of a washing machine suspension system

Thomas Nygårds, Viktor Berbyuk

Department of Applied Mechanics, Chalmers University of Technology, Göteborg, Sweden
thomas.nygards@chalmers.se, viktor.berbyuk@chalmers.se

Abstract

The washing machine is a well known home appliance which is used at least weekly, in some cases daily, by almost every family. The importance of the task the machine performs combined with the variety of available machines has made competition between manufacturers harder and harder. Among the strongest drive forces in the field of washing machine development are the capacity increase together with reduction of energy consumption. Since the first machines with spin-drying ability the washing machine has had a reputation of being noisy and causing vibrations. A soft suspension improves the vibration isolation and reduces the vibration output, but the severity of the vibration problem can be increased with the bigger tub volume that a machine with higher capacity demands. As the same outer standard dimensions of the machine housing must be preserved, a stiffer suspension might be needed to keep the tub from hitting the housing at when passing the critical spin speed. Hence there are conflicting criterias to be dealt with.

This paper focuses on several aspects of vibration dynamics in washing machines: the capacity maximization through the study of tub movement, the vibration output from the machine to the surroundings, and the “walking” tendency of the system. A computational model of a washing machine with bottom mount suspension has been built in Adams/View from MSC. Software, based on production drawings. Experimental data was used for validation of mathematical and computational models of functional components of the system as well as for the model of the complete washing machine. The models of the functional components have been parameterized and are used for suspension optimization in a computer cluster. Three objective functions related to kinematics and dynamics of washing machines have been defined and a numerical algorithm has been created to solve Pareto optimization problems. The algorithm is a genetic algorithm built around Matlab’s subroutine “gamultiobj.m” and executed on an in-house developed computer cluster with possibility of parallel computing of Adams/View models. The results are presented as optimized parameter values of suspension functional components, in this case bushings with respective Pareto fronts. The focus has been set on delivering couplings between parameter values and performance trade-offs in terms of objective functions to facilitate parameter tuning. The obtained optimization results have successively been used in the development of a novel washing machine which will go into production after the summer 2010.

Keywords: Washing machine suspension, Vibration dynamics, Pareto optimization, Parallel computing, Adams/View

1. Introduction

Most consumers associate washing machines with vibration and noise. The vibration problem in washing machines originates from a rotating imbalance. Imbalance is caused by unevenly distributed load inside the washing machine’s inner tub or drum. A load which can vary from for example a pair of dirty jeans to an entire week’s accumulated laundry which has to be pressed into the drum to enable the closing of the tub door. Machines are constructed in such a way that a wide range of different load weights and fill volumes are accepted. Load capacities of washing machines are given in kilograms of dry load and in addition to the dry load up to 400% extra weight is added when water is added during washing. This means that when the machine is operated with maximum load the mass of the suspended tub system can become close to doubled compared to the case of minimal load. Hence, this load flexibility puts high demands on the tub suspension system of the washing machine. Research has been performed on washing machines to analyze the suspension system to gain understanding on how to prevent “the oscillatory walk of washing machines”, or simply “walking” i.e. shift position [1, 2]. But when the electronic imbalance control was incorporated into washing machines, in the late 80’s - early 90’s, the problem became smaller and went out of focus of the washing machine research field. However, the standard of people’s homes is increasing, home design is more important and the installation locations of washing machines are becoming more varied. In some countries the default installation position being on a rough cement floor of a basement or on linoleum flooring is challenged with installations in esthetically designed rooms [3], inside closets, under kitchen sinks, sometimes on top of highly polished wooden floors. The electronic imbalance control can be tuned to make sure that only loads with small imbalances can perform spinning, but to not risk that no high speed spinning is performed the limit cannot be set to low. As a consequence of this the customers still experience that

their washing machines “walk”. Walking can be prevented if the machine is fixated to the floor structure. In commercial or community installations this is often practiced. Installations in private houses are seldom done in this way, due to difficulty of such installation and to the destructive modifications needed to be done to the floor. The washing machine, to which the methodology of this paper is applied, is a front loaded washing machine with bottom mount suspension. The tub is suspended on springs which together with a friction element based damper are incorporated inside four struts. A picture of the washing machine system is shown in figure 1.

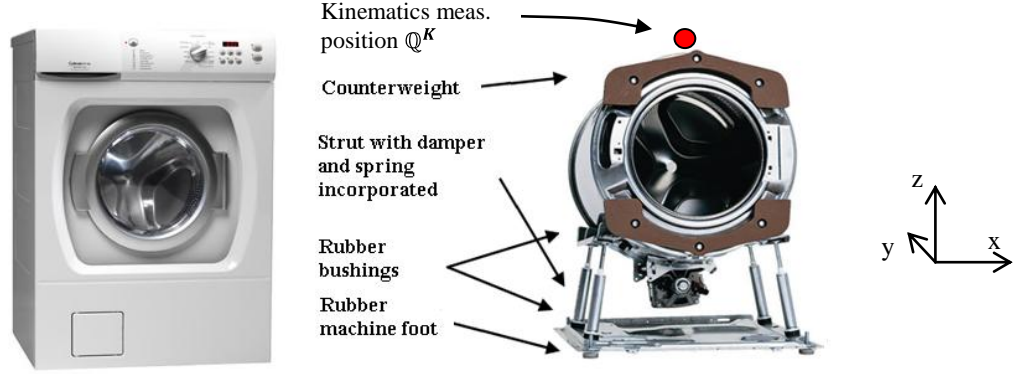


Figure 1: The studied washing machine with its internal parts

The methodology of vibration dynamics analysis and functional component optimization described in this paper applied on a computational model, describing the dynamics of the washing machine depicted in figure 1. The model is a rigid multibody system representing the washing machine by including 13 dof. The model is built with computer drawings as inputs for geometric and inertia parameters, which together with individually modeled functional components like dampers, bushings, springs and constraints are implemented in Adams/View. Generally, the model can be described by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t, \mathbf{p}, \mathbf{d}, \mathbf{l}, \mathbf{u}) \quad (1)$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}, t, \mathbf{p}, \mathbf{d}, \mathbf{l}, \mathbf{u}) \quad (2)$$

$$\mathbf{x}(0) = \mathbf{x}_0 \quad (3)$$

Here \mathbf{x} is the state vector of the system, \mathbf{p} is the vector of structural parameters, \mathbf{d} is the vector of design parameters subject to optimization, \mathbf{l} is the vector of load parameters, \mathbf{u} is the vector of external control stimuli, \mathbf{y} is the output vector, and \mathbf{f} and \mathbf{g} are given vector functions. Further description of the mathematical and computational models is given in [4,5,6].

In this paper, first basic properties of the washing machine as a softly suspended rotor system are presented. Later the general washing machine vibration problem and some general vibration performance measures are stated. Based on these statements the formulation of a specific bi-objective optimization problem treated in this paper is given. The solution to the problem giving the engineer a trade-off solution between the different objective functions is presented.

2. Washing machine dynamics

The washing machines considered in the scope of this paper belong to the horizontal-axis washing machines. This means in the context of washing machines that the drum axis is oriented in a plane roughly parallel to the floor which the machine stands on.

The motion of the center of mass of the tub of a washing machine is periodic under constant operational conditions, i.e. steady state. The motion at low spinning speed, when centrifugal force is low, can have different shape depending on the type of suspension used. But as the washing machine as a mechanical system is a softly suspended rotor an oval shaped motion can be anticipated for simple static unbalance loads at higher speeds. In figure 2 the trajectory of a point on the tub \mathbf{Q}^K (see figure 1) in the x-z plane when the drum is accelerated from 0 to 400 rpm during 6 seconds with a large imbalance placed in the front end of the drum is shown. The motion is divided in into segments in time to more clearly show the different motion shapes of the point. The time is indicated with a color bar at the top of each plot changing from blue to red with increasing time. The kinematics of the tub is calculated with the dynamic model Eq.(1), Eq.(2), Eq.(3). The vertical forces output from two of the machine feet are shown to the right for the same load case and spin speed profile.

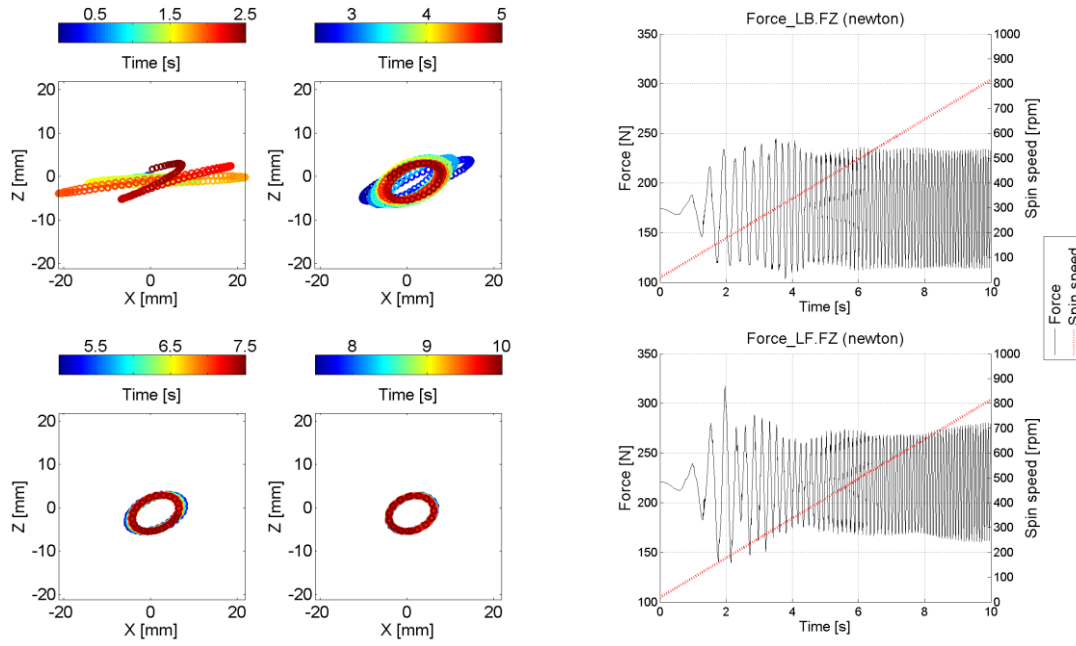


Figure 2: *Left:* The motion in the x-z plane of a point on the tub of the washing machine
Right: The vertical force propagated through the feet of the left side of the machine and given drum speed excitation

3. The optimization problem

The outer size of a washing machine often is standardized to correspond to kitchen and bathroom fittings, typically 600 mm wide and deep. With a fixed limit on the outer size of the housing the physical upper limit of volumetric capacity is set. Then, of course not all volume inside the outer housing can be used for the drum; some must be used for auxiliary components like motor, electronics, water tubing, detergent box, suspension etc. To maximize the volumetric capacity of a machine, within a given space which can be used for the drum, a trade off must be done between the size and the motion of the tub. Moving the tub outside this given space will mean collision between components which might lead to more noise or to destruction of components in the most severe case. Motion of the tub can be reduced by lowering of the unbalanced load with for example better load distribution, counter balancing techniques, or by stiffening (dynamic) of the suspension system when passing critical resonances. Stiffening of the suspension can be done with active or semi active solutions, like magneto-rheological solutions [7], with nonlinear passive solutions like gap dampers [8, 9] or with conventional passive dampers. Considering the conventional passive solution a reduction of tub movement will lead to an increased transmissibility of forces to the floor structure as the suspension system stiffens. Therefore a trade-off also exist between motion of the tub and vibration output.

In this paper three criteria of high importance in washing machine dynamics are presented. The first serves to measure the motion of the tub. Minimizing this cost function will work towards keeping the tub free from hitting the housing during washing and/or possibility to increase the capacity of the washing machine through increased tub size. The second is to measure the vibration output of the machine. Minimization of this cost function will reduce the vibration propagation, originating from the rotating imbalance, through the washing machine suspension. Isolation of the source of vibration with help of a good suspension will lead to small vibration impact on surroundings. The third criterion is to prevent the washing machine from moving from its installation position. Minimizing this cost function will enable secure installation of a machine on more slippery floors, and/or reduce risk of walking.

3.1. Kinematic objective

The intuitive desire when it comes to tub kinematics is to have as little motion of the whole tub as possible. So, based on this the distance between the tub and the housing and its components should be measured all over the tub surface. This is not easy to do in practice and in the computational model it is not easy either, as the geometries of the tub and housing critical components are difficult to parameterize due to their complex shape. But, the motions in all directions are not necessarily critical to keep low, i.e. in some directions the motion might be small regardless

of imbalance, and in some directions there are empty spaces which cannot be used to increase capacity but maybe could “be used” for motion. With this in mind several critical points \mathbb{P}_k , and respective critical directions \mathbf{v}_k were identified, $k=1,2,3,\dots,10$. Typically the points were selected based upon if the engineers, testers or consumers have had experienced problems with collision at these points during operation. In addition to these points some additional points were selected to reflect remaining directions not covered directly by experienced problems but still deemed realistic as potential areas of collision. In figure 3 a possible definition of such points and directions is shown.

Each point \mathbb{P}_k describes the location of a critical point on the moving tub. Coupled to the point there is a vector \mathbf{v}_k which describe a critical direction. Motion of the tub at point \mathbb{P}_k in the direction of \mathbf{v}_k is described by $X_k(t) = \langle \mathbf{r}(\mathbb{P}_k), \mathbf{v}_k \rangle$, i.e. the scalar product between the point \mathbb{P}_k position vector $\mathbf{r}(\mathbb{P}_k)$ and directional vector \mathbf{v}_k . Let X_k^{\max} denote the movement margin of point \mathbb{P}_k . This constitutes the maximum admissible motion for collision avoidance. The values of \mathbb{P}_k , \mathbf{v}_k and X_k^{\max} have been defined, when possible, by measurements performed on a real machine and later validated in the CAD model. If a physical measurement was not possible, measurement was performed directly in the CAD model.

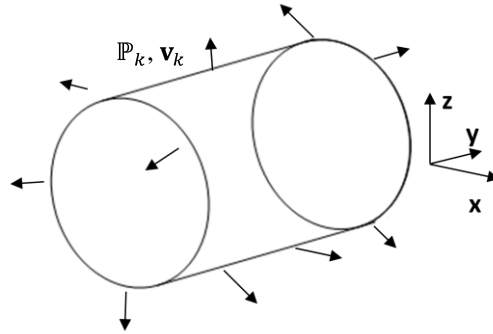


Figure 3: An illustration of a definition of critical movement directions

Taking into account the definitions above the kinematic objective to be minimized is defined as follows

$$\mathcal{F}_K = \max_k \left(\max_t (X_k(t) - X_k^{\max}) \right), \quad k = 1, 2, 3, \dots, 9, t \in [0, T] \quad (4)$$

Typically for a good suspension and admissible operational conditions, no collision occurs and $\mathcal{F}_K < 0$.

3.2. Vibration output dynamic objective

Vibration is a problem tightly coupled to washing machines, as stated earlier. One of the challenges when it comes to washing machines is to define how the impacting vibrations should be measured. Apart from measurement directly on vibrating components of the machine a few attempts have been made to classify the amount of vibrations that a washing machine causes on the surroundings. Such examples are the method for testing developed for Consumers Union in USA and the method previously used by Swedish Consumer Agency [10]. Both methods are based on measurement of accelerations at specified points in its surroundings and include a standardized floor (not the same between the methods). In Sweden today, the only vibration related performance classification performed is a sound pressure level measurement. The problem with vibration impact measurement on a floor or another external object is that the properties of the external object influence on the measurement value. This is perhaps one of the reasons to that a standard international measurement method not yet is established.

Furthermore, incorporation of a method based on measurement of surroundings in a computer model to be used for optimization might be unnecessary complicated. If the dynamics of a floor also must be calculated more computational resources are needed, apart from the risk of that the incorporated floor model does not represent the actual floor to a sufficient extent. Also the aspect of over-fitting of the washing machine components to a particular floor must be considered in such a method.

To skip the complexity of floor modeling the transmitted force to a fixed base is measured directly. In earlier papers published by the authors a test rig was presented [4,6]. In this test rig it is possible to measure the transmitted force in vertical direction. To enable the use of same cost function for measurement and simulations only the vertical direction was used as in data. In addition, it shall be stated that the vertical direction is the dominating when it comes to transmitted force, both in absolute value and in magnitude of oscillations. Second in magnitude is the lateral force when it comes to front loaders and the longitudinal force when it comes to horizontal axis top loaders.

The dynamic objective to be minimized is the sum of the RMS-values of the dynamic vertical forces F_i^z , $i=1,2,3,4$ at the four feet of the machine during the simulation time T . To more clearly enhance the dynamic part of the forces, which are the reason to vibrations, the weight of the machine is removed by removing the forces when no motion of the drum occurs and when the machine is in static condition. This is assumed to occur at time $t=0$. Based on this the suggested dynamic cost function is written as follows

$$\mathcal{F}_D = \sum_{i=1}^4 \sqrt{\frac{1}{T} \int_0^T \left(F_i^z(t) - F_i^z(0) \right)^2 dt} \quad , \quad F_i^z(t) = \langle \mathbf{R}_i(t), \mathbf{e}_z \rangle \quad (5)$$

Here \mathbf{R}_i is the reaction force from the floor to the machine vibration at foot i , $i=1,2,3,4$ and \mathbf{e}_z is a unit vector in the vertical direction.

3.3. Walking objective

The objective stated in equation Eq.(5) has the purpose of minimizing the vibration output by minimizing the transmitted vertical forces between the machine and the floor. The vertical force is related to walking but the problem of walking is not addressed with the above objective. It might even have the opposite effect if the vertical forces are minimized with the drawback of increased lateral forces. When the machine walks, a foot slides relatively to the floor. Sliding can occur when the friction force is too low to keep the foot in place, but also if an underlying object (drip pan, carpet etc.) is placed between the machine and the floor. This underlying object can produce a more slippery contact towards the machine or towards the floor than the machine foot contact would be directly towards the floor. This condition can be modeled as multiple serially connected friction contacts where the most slippery will slip first. Friction is a complex nonlinear phenomenon and still is an area for research. Let us assume that we are dealing with Coulomb dry friction which is modeled according to

$$F_{fr} \leq \mu F_N \quad \text{or} \quad \frac{F_{fr}}{F_N} \leq \mu \quad (6)$$

Here F_{fr} is the friction force, μ is the coefficient of static friction and the normal force of the floor is F_N . The transition to sliding occurs when equality in Eq.(6) is reached, hence the highest margin to walking is acquired when F_{fr}/F_N is minimized and $F_{fr}/F_N < \mu$. The corresponding friction coefficient which will keep foot i from losing its frictional force can be written as

$$\mathcal{F}_{W,i}^i = \max_t \left(\frac{\| \mathbf{F}_i^{xy}(t) \|}{F_i^z(t)} \right), \quad i = 1,2,3,4, t \in [0, T] \quad (7)$$

$$\mathbf{F}_i^{xy}(t) = \langle \mathbf{R}_i(t), \mathbf{e}_x \rangle \mathbf{e}_x + \langle \mathbf{R}_i(t), \mathbf{e}_y \rangle \mathbf{e}_y$$

Here \mathbf{e}_x and \mathbf{e}_y are unit vectors in the lateral and longitudinal directions respectively and $\|\cdot\|$ is the Euclidian norm of a vector.

Equation Eq.(7) has only frictional meaning if $F_i^z(t) > 0$. For $F_i^z(t) \leq 0$ an adhesive surface contact or similar is needed to keep the foot from slipping. From Eq.(7) the walking objective which will measure the surface friction coefficient needed to prevent any foot from losing the grip can be formulated according to the following equation

$$\mathcal{F}_{W,i} = \max_i (\mathcal{F}_{W,i}^i) \quad (8)$$

3.4 Optimization problem formulation

Let \mathbf{F} be the m -dimensional vector objective function which determines the mapping $\mathbb{R}^n \rightarrow \mathbb{R}^m$ from the n -dimensional space of design parameters \mathbf{d} into the m -dimensional space of quality factors.

The design vector $\mathbf{d} \in \mathbb{R}^n$ is usually constrained by $\mathbf{d} \in \mathbf{X}_d \in \mathbb{R}^n$, where \mathbf{X}_d can be defined by algebraic or/and differential constraints. A multi-objective optimization problem is stated as follows:

Problem A. Determine the set of vectors of structural parameters \mathbf{d}_* , such that

$$\mathbf{F}(\mathbf{d}_*) = \min_{\mathbf{d}} \mathbf{F}(\mathbf{d})$$

The solution of *Problem A* is called Pareto optimal. The Pareto optimal solution \mathbf{d}_* is defined as such that $\mathbf{d}_* \in \mathbf{X}_d$ and that there is no $\mathbf{d}_* = \mathbf{d} \in \mathbf{X}_d$ with $\mathbf{F}(\mathbf{d}) \leq \mathbf{F}(\mathbf{d}_*)$ for $i=1,2,\dots,m$ with strict inequality for at least one

$i, 1 \leq i \leq m$. The vector function $\mathbf{F}(\mathbf{d}_*)$ determines the Pareto front and the set of all points $\{\mathbf{d}_*\}$ is termed Pareto set for *Problem A*.

As stated earlier, a washing machine has to be able to handle different amounts of load and different imbalances. To reflect these different operational conditions two critical load cases with respective drum rotational excitation schemes are defined as follows:

1. Constant load of 1 kg placed in the front of the drum whilst spinning up to 800 rpm with a gradient of 80rpm/s.
2. Constant load of 0.3 kg placed in the middle of the drum whilst spinning up to 1600 rpm with with a gradient of 80rpm/s.

These conditions constitute operational condition denoted by $\mathbf{l}_1, \mathbf{u}_1$ and $\mathbf{l}_2, \mathbf{u}_2$. The bi-objective optimization problem of washing machine vibration dynamics on a given set of operational conditions subject to kinematic constraints is stated as follows:

Problem B. Determine the vector of structural parameters \mathbf{d}_* and corresponding state vectors $\mathbf{x}_*(t)$ which satisfy the variational equation

$$\min_{\mathbf{d} \in X_d} \mathbf{F}[\mathbf{d}, \mathbf{l}, \mathbf{u}, \mathbf{x}(t)] = \mathbf{F}[\mathbf{d}_*, \mathbf{l}, \mathbf{u}, \mathbf{x}_*(t)]$$

subject to the differential equations of motion Eq.(1), Eq.(2), Eq.(3) and restrictions $\mathbf{B}_l \leq \mathbf{d} \leq \mathbf{B}_u$. Here $\mathbf{F} = [\mathcal{F}_K, \mathcal{F}_D]^T$, $\mathbf{l} = [\mathbf{l}_1, \mathbf{l}_2]$ and $\mathbf{u} = [\mathbf{u}_1, \mathbf{u}_2]$ where $\mathbf{l}_j, \mathbf{u}_j$ are vectors of input parameters together defining the operational condition j , ($j=1,2$).

The solution to the problem is a Pareto front determined by $\mathbf{F}[\mathbf{d}_*, \mathbf{l}, \mathbf{x}_*(t)]$ which describes optimal trade-off solutions for the two objectives \mathcal{F}_K and \mathcal{F}_D . Coupled to the Pareto front is a set of vectors $\mathbf{D}_* = \{\mathbf{d}_*\}$ of structural parameters \mathbf{d}_* which each is a vector of optimal parameters for a point on the Pareto front.

The parameters considered to constitute \mathbf{d} were selected to be the linear stiffness and damping of the lower rubber bushing. The upper and lower restrictions on the bushing stiffness parameter were selected from static experiments performed on available bushings with same geometry ranging from 30 to 80 shore in hardness. The damping parameter restrictions were taken from $\pm 500\%$ of the identified damping value of the current washing machine model.

4. Implementation in computer environment

The objective functions which are minimized when problem A is solved use response from a simulated computational model describing the washing machine dynamics. The model is implemented in MSC.Adams/View [11]. A Matlab-Adams\View communication interface which was constructed earlier enables a function in Matlab to start simulations in parallel and evaluate cost functions as the simulations are completed. This interface is called by “gamultiobj.m” with a generation of solution candidates. The interface returns vectors with cost function values, and returns the constraint errors which were calculated from the simulated data if the bi-objective optimization is subjected to kinematic or dynamic constraints. The chosen optimizer is a so called hybrid optimizer which divides the optimization into two steps. First the Matlab function “gamultiobj.m”, which is a multi objective optimizer based on genetic algorithms, is used to find a draft Pareto front in the given search space. After the genetic part of the optimization is completed a gradient based optimization routine (“fgoalattain.m”) is applied on the points of the Pareto front to try to push the front even further towards optimality. The reason for selection of a genetic algorithm as a first step is that it is suitable for parallel computation. With the selected algorithm every step comprises not only one solution candidate but various. All of these are made available for evaluation in parallel by the algorithm. Note that to calculate the response of a solution candidate more than one simulation is needed, as individual simulations are needed for all different operational conditions. Unfortunately the function “fgoalattain.m” only works in parallel if called in parallel with Matlab’s own parallelization toolbox and does not estimate gradients in parallel, i.e. call for evaluation of several points at the same time, when used as a hybrid function by “gamultiobj.m”.

More details on the Matlab-Adams\View communication interface can be found in [12].

5. Results and analysis

The resulting Pareto front is presented in figure 4. In the figure also results for the simulated parameter configurations that have functional values within the range of the plot axes are presented. It can clearly be seen that a conflict exist between the two objectives. But, the conflict is only real if the washing machine is designed with some optimal parameters $\mathbf{d}_1 \in \{\mathbf{d}_*\}$ which give optimal performance for some given priority between the objectives.

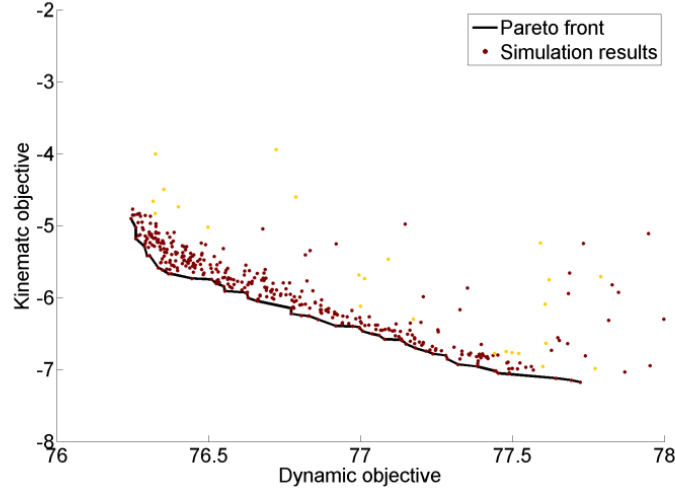


Figure 4: The Pareto front and results of some evaluated parameter configurations

This means that if the priority between the objectives changes, then a worsening of one of the objectives is unavoidable. If the machine is designed with parameters $\mathbf{d}_2 \notin \{\mathbf{d}_*\}$ an improvement of both objectives is possible. In figure 5 the resulting motion $X_{k_1}(t)$, $X_{k_2}(t)$ are plotted for five different suspension configurations distributed evenly along the Pareto front. The difference in peak to peak motion is exceeding 30% between the worst and the best suspension configuration with respect to the kinematic objective.

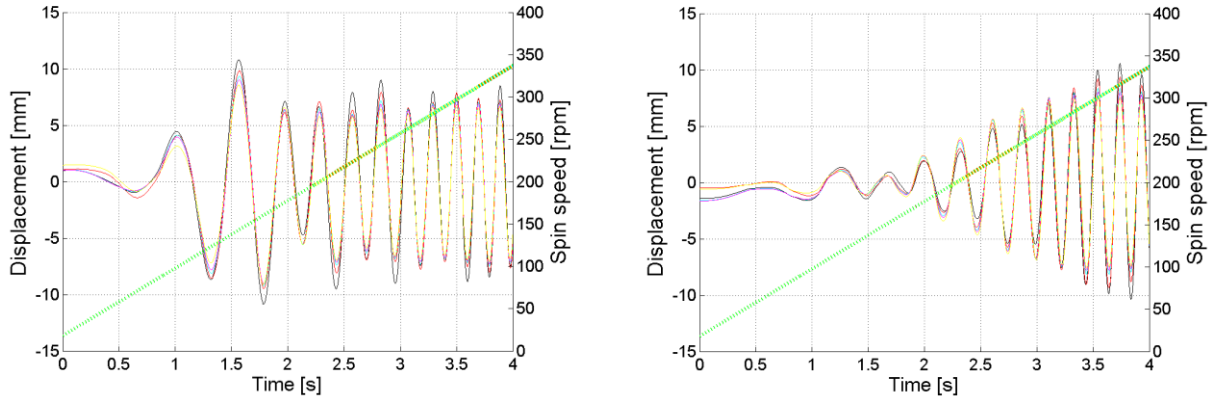


Figure 5: The kinematics of two of the tub critical points for five different suspension configurations taken from the Pareto front

The objective function space which represent possible performances with respect to Eq.(4) and Eq.(5) of the washing machine are coupled to the suspension parameters allowed to vary, which constitutes the parameter space \mathbf{X}_d for the considered problem. This coupling is generally enough to use to select the parameters which give desired performance. It is just to follow the coupling from a point in the objective function space to the parameter space and implement the corresponding machine suspension.

The requirements on performance are seldom static for a product, not even for a washing machine. A Pareto optimal solution gives the answer on what optimal parameters to select for another prioritization of the requirements on performance of operation. To use the results of an optimization efficiently it is good if a trend in the objective function values can be found with respect to varying parameter data. In this way an engineer can know how much to tune each parameter if a change of the priority amongst the cost functions is desired. To find the trend for the subspace \mathbf{D}_* of the parameter space which is mapped to the Pareto optimal solution, continuous polynomials were mapped against the respective variable subspace. In figure 6 a-d the fitted polynomials are shown together with the points on the Pareto front. The polynomials represent the rubber bushing damping correlation to the cost functions relatively well, but it can be seen in the case of stiffness correlation cannot be assumed here. Generally the coupling is not one to one, that is, several different parameter sets can give the same values of performance.

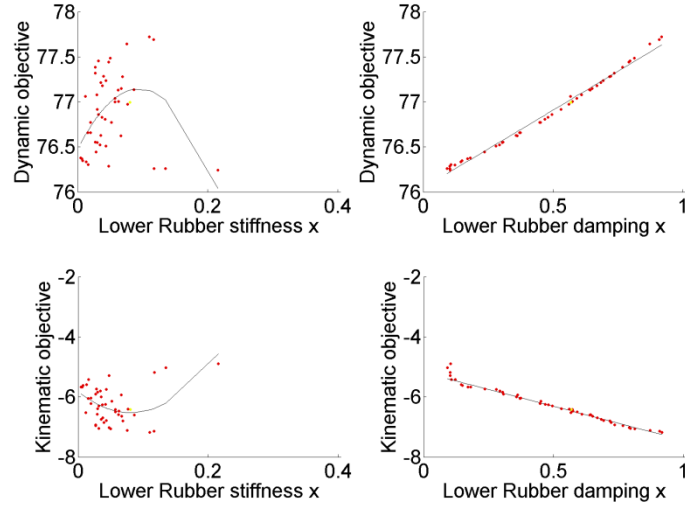


Figure 6 a-d: The values of the points on the Pareto front plotted against corresponding parameters

There are often practical problems with incorporation of a solution found from computing a model into a physical prototype. For example, it is more a rule than an exception that a found solution of an optimization problem will require change of all parameters to reach to the optimum configuration of a system. This might be acceptable if it is the first time a prototype is built. But when it comes to optimization of already existing production model as few changes as possible are desired. Each change is usually associated with a cost increase, for example change of production equipment or addition to the number of components to have in stock. It is therefore interesting to know how much the solution is affected if other component values the ones the optimization found are used. Of course, if a component is expensive to change it can be set fixed before the optimization is run, but if the effect on the cost functions is unknown, it might lead to a missed opportunity for a performance increase. In essence it could be better to do the elimination afterwards. If the change in solution is small the sensitivity of optimum with respect to the fixed parameter can be said to be small.

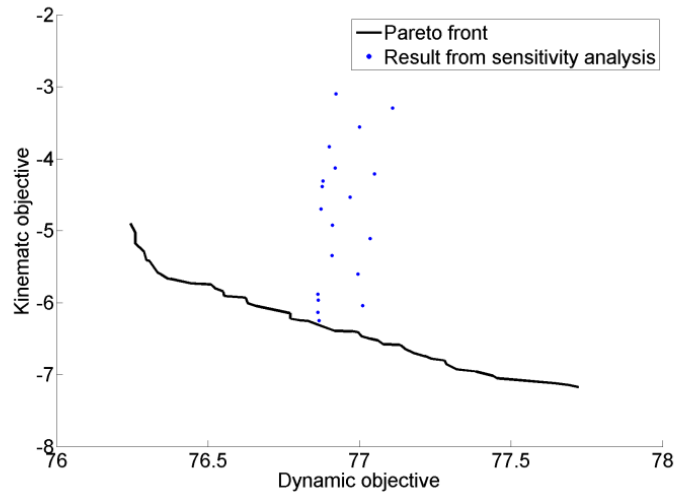


Figure 7: The found Pareto front and results of some evaluated parameter configurations

To analyze the sensitivity of the solution with respect to the optimized variables, a point corresponding to structural parameters $\mathbf{d}_1 \in \{\mathbf{d}_*\}$ on the Pareto front $\mathbf{F}[\mathbf{d}_*, \mathbf{l}, \mathbf{x}_*(\mathbf{t})]$ was selected. All but one variable, called $d_1 \in \mathbf{X}_d$, were fixed. The variable d_1 was varied between its max and min limits and the values of the objective functions were calculated. Generally, some of the resulting function values can coincide with the already found Pareto front, indicating small sensitivity of the found solution to the varied variable.

In this paper the rubber damping parameter was held fixed with a value corresponding to the normalized value 0.5. The other parameter (rubber stiffness) was then varied within its limits. The resulting functional values are plotted

in figure 7 together with the Pareto front found earlier. To clarify the effect of the stiffness on the particular objectives in figure 8 the stiffness values coupled to the points in figure 7 are plotted. It is clear that the sensitivity of the dynamic objective with respect to stiffness in this case is small. Simulations have also shown that the ripple of the curve probably can be lowered with reduced step size and smaller tolerance of the error during simulation of the model. The sensitivity of the kinematic objective with respect to stiffness is not small which can be seen in the right plot of figure 8. The margin to collision is in fact halved for the worst selection of stiffness.

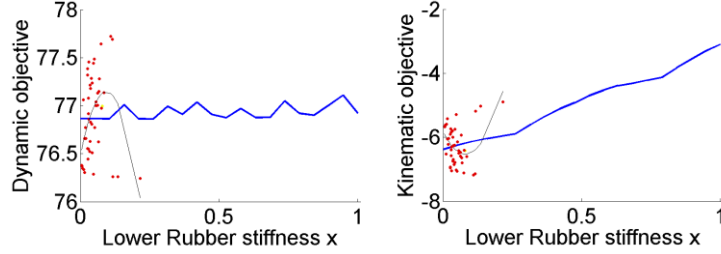


Figure 8: The effect of stiffness variations around a point in the Pareto set

In this paper the resulting value of the walking objective is only used for analysis and not for optimization. It is of course possible to use it directly as an objective to be minimized. But, the objective can also easily be transformed to a constraint if the critical friction coefficient μ_c of the most slippery floor/foot/drop pan combination for which the machine is needed to perform is given. This critical coefficient can for example be determined and used by a manufacturer to establish recommendations for safe installation of their machine. The dynamic constraint which is designed to prevent slip on any foot can then be written as

$$\mathcal{F}_{Wt} \leq \mu_c \quad (9)$$

The results from calculation of the walking objective Eq.(8) from the response of a simulation of the model with a suspension configured according to the parameter points in the Pareto set \mathbf{d}_1 for the operational condition $j=1$ are shown in figure 9.

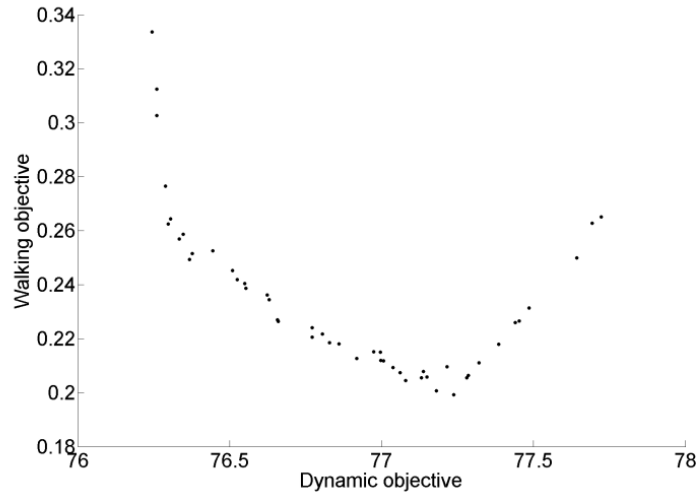


Figure 9: The value of the walking objective for the parameter configurations on the Pareto front plotted against the dynamic objective

Measurements on a real machine placed on a slippery floor has shown that the static friction coefficient between sliding surfaces under the machine can be so small as 0.14 under the most slippery conditions the machine has to operate on. As all of the parameter configurations found results in a value of the walking objective which exceeds this number it may indicate that walking is a problem. The walking objective Eq.(8) is formulated in such a way that if slip occurs at only one foot the objective function will produce a high value. It is however not necessary the case that the machine will move as the friction force from the remaining feet might be sufficient to hold the machine in place. Further study on the subject may be needed.

6. Conclusions and future work

A methodology for bi-objective optimization of the dynamics of a washing machine model has been presented. An example of a bi-objective optimization problem is solved for a washing machine of bottom mount suspension type and the results have been presented and analyzed on the form of Pareto fronts together with couplings to design parameter space. The existence of a trade-off solution for collision avoidance and vibration isolation in a washing machine of this type is established. A sensitivity analysis of the impact of the stiffness of the lower bushing of the strut on the respective cost functions has been done. It showed that for a given damping parameter belonging to the Pareto set, the stiffness parameter can affect the kinematic objective with almost 50% but not the dynamic objective. The walking phenomenon of washing machines has been studied for the system with obtained Pareto set as design parameters by using a walking criterion related to friction forces.

In the future the walking criterion formulation will be extended to cover the case where one foot slips but the friction forces of the remaining feet are large enough to keep the washing machine from walking.

Also, an adapted version of the Matlab multi-objective gradient based optimization function “fgoalattain.m” will be developed to enable parallel evaluation of gradients in the developed Matlab-Adams\View communication interface.

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References

- [1] D.C. Conrad:, *The fundamentals of automatic washing machine design based upon dynamic constraints*, Purdue University, 1994, 261 pages; ISBN 9780591345728.
- [2] E. Papadopoulos, I. Papadimitriou: Modeling, Design and Control of a Portable Washing Machine during the Spinning Cycle, *Proceedings of the 2001 IEEE/ASME International Conference on Advanced Intelligent Mechatronics Systems (AIM 2001)* pp. 899-904
- [3] Washington Times Webpage “Cover story: Laundry area climbs out of basement”
<http://www.washingtontimes.com/news/2010/may/14/cover-story-laundry-area-climbs-out-of-basement/>
(Accessed June 3, 2010)
- [4] T. Nygård, V. Berbyuk: Dynamics Of Washing Machines: MBS modeling and experimental validation, *Proceedings of the Multibody Dynamics 2007, ECCOMAS Thematic Conference*, Milano, Italy, 25–28 June 2007.
- [5] T. Nygård, V. Berbyuk, A. Sahlén: Modeling and Optimization of Washing Machine Vibration Dynamics. *Proceedings of the 9th International Conference on Motion and Vibration Control (MOVIC 2008)*, September 15-18, 2008, Technische Universität München, Munich, Germany.
- [6] T. Nygård: *Modeling and Optimization of Washing Machine Vibration Dynamics during Spinning*. Thesis for Licentiate Engineering 2009:1, Department of Applied Mechanics, CHALMERS, 2009
- [7] C. Spelta, et al.: Control of magnetorheological dampers for vibration reduction, *Journal of Mechatronics*, Volume 19, issue 3, p.410-421, Apr 2009
- [8] Suspa website: “Suspa.com - RD 18 FL” <http://www.suspa.com/index.php?id=786> (Accessed May 5, 2010)
- [9] Aweco Appliance Systems website:
<http://www.aweco.de/geschaeftsfelder/hausgeraete/home-laundry/waschmaschine>. (Accessed May 5, 2010)
- [10] Testlab konsumentverket, Provningsinstruktion I-2200 P, Utg 1, Rev A, 1998.
- [11] Msc. Software Webpage: “Adams for Multibody Dynamics”
<http://www.mscsoftware.com/Contents/Products/CAE-Tools/Adams.aspx> (Accessed June 4, 2010)
- [12] T. Nygård, V. Berbyuk: An Adams/View-Matlab Computational Interface for Clustered Optimization of Washing Machines, *Proceedings of the 22nd Nordic Seminar on Computational Mechanics (NSCM)*, 22-23 October 2009, Aalborg University, Aalborg, Denmark, DCE Technical Memorandum No. 11, ISBN 1901-7278